Quantum Phase Estimation properties:

**Let:** () is true if is a unitary operator, and is an eigenstate of

returns the eigenvalue of the eigenstate with respect to matrix

applies quantum phase estimation with a hardcoded matrix with input qubits in the state with estimation qubits.

All of the below are assuming that enough estimator qubits are used for the algorithm to work.

* **Inputs**:
* **Precondition**:
* **Operation**:
* **Postcondition**:
* **Inputs**: ,
* **Precondition**:

* **Operation**:

* **Postcondition**:
* **Inputs**: ,
* **Precondition**:

* **Operation**:

* **Postcondition**:

* **Inputs**: ,
* **Precondition**:

* **Operation**:

* **Postcondition**:

(NOT SURE ON THIS BOTTOM ONE)

* **Inputs**: ,
* **Precondition**:

* **Operation**:

* **Postcondition**:

[Summing two eigenvectors with the same eigenvalue, yields another eigenvector with the same eigenvalue.](https://math.libretexts.org/Bookshelves/Linear_Algebra/Map%3A_Linear_Algebra_(Waldron_Cherney_and_Denton)/12%3A_Eigenvalues_and_Eigenvectors/12.03%3A_Eigenspaces#:~:text=In%20simple%20terms%2C%20any%20sum,%CE%BB%20is%20called%20an%20eigenspace.)

* **Inputs**: ,
* **Precondition**:

* **Operation**:

* **Postcondition**:

[Summing two eigenvectors with different eigenvalues does not yield an eigenvector](https://math.stackexchange.com/questions/2251572/sum-of-two-eigenvectors-for-different-eigenvalues)

* **Inputs**: ,
* **Precondition**:

* **Operation**:

* **Postcondition**:

Proofs:

**“If we apply a unitary operation on an eigenvector, the state does not change.”**

Let be a unitary matrix with eigenvector , and eigenvalue .

Consider the eigenvector equation:

It is easy to see that the application of a unitary matrix only scales the eigenvector but does not apply a rotation.

**“If we apply a unitary operation to a state that is not an eigenvector, the state does change.”**

Proof by contradiction:

Suppose that “If we apply a unitary operation to a vector that is not an eigenvector, the state does **NOT** change.”

That is: where an eigenvector of .

However, this contradicts with eigenvector equation that assumes that is an eigenvector of

**“Eigenstates with different eigenvalues result in different phases to be estimated by QPE.”**

QPE estimates the phase from unitary with eigenvector and eigenvalue where .

, (all eigenvalues lie on the unit circle).

We need to prove that is ***injective*** for , that is:

Direct proof:

Assume: ,

Substitute formula: : = ,

Apply Euler’s Formula:

Complex numbers are equal if their imaginary and complex components are equal thus:

Both conditions are only simultaneously satisfied when within the domain of thus is ***injective*** within the domain of ***.***